Thomas Weatherby

Theo III: 4 Eigenschaften des Skalarproduktes und weitere „Operatorgymnastik“

Wiederholung vom letzten Mal

- Was haben wir letzte Woche gemacht?

Eigenschaften des Skalarproduktes

1. $\langle x|x\rangle \geq 0$
2. $\langle x|x\rangle = 0 \ \leftrightarrow x = 0$
3. $\langle x|y\rangle = \langle y|x\rangle^*$
4. $\langle x|\lambda y\rangle = \lambda\langle x|y\rangle$
5. $\langle x|y + z\rangle = \langle x|y\rangle + \langle x|y\rangle$

Eigenschaften des Skalarproduktes

Beweise $\langle \lambda x | y \rangle = \lambda^* \langle x | y \rangle$

Eigenschaften des Skalarproduktes

Beweise $\langle x + z | y \rangle = \langle x | y \rangle + \langle z | y \rangle$

Operatoren

Beweise für zwei hermitische Operatoren, $\Omega_1 \; und \; \Omega_2$, dass $[\Omega_1, \Omega_2]$ anti-hermitisch ist.

How is a wave-function $\psi(x)$ written in Dirac's notation? What's the physical significance of the complex number $\psi(x)$ for given x?

Operatoren

Let Q be an operator. Under what circumstances is the complex number $\langle a|Q|b\rangle$ equal to the complex number $\langle b|Q|a\rangle^*$ for any states $|a\rangle$and $|b\rangle$?

Operatoren

Let Q be the operator of an observable and let $|\psi\rangle$ be the state of our system.

a. What are the physical interpretations of $\langle\psi|Q|\psi\rangle$ and $|\langle q_n|\psi\rangle|^2$, where $|q_n\rangle$ is the n^{th} eigenket of the observable Q and q_n is the corresponding eigenvalue?
b. What is the operator $\sum_n |q_n\rangle\langle q_n|$, where the sum is over all eigenkets of Q? What is the operator $\sum_n q_n|q_n\rangle\langle q_n|$?
c. If $u_n(x)$ is the wavefunction of the state $|q_n\rangle$, write down an integral that evaluates to $\langle q_n|\psi\rangle$.

Operatoren

What does it mean to say that two operators commute? What is the significance of two observables having mutually commuting operators? Given that the commutator $[P, Q] \neq 0$ for some observables P and Q, does it follow that for all $|\psi\rangle \neq 0$ we have $[P, Q]|\psi\rangle \neq 0$?

Let $\psi(x,t)$ be the correctly normalised wavefunction of a particle of mass m and potential energy $V(x)$. Write down expressions for the expectation values of:

$(a)\ x$;

$(b)\ x^2$;

(c) the momentum p_x;

$(d) p_x^2$;

(e) the energy.

What is the probability that the particle will be found in the interval (x_1, x_2)?