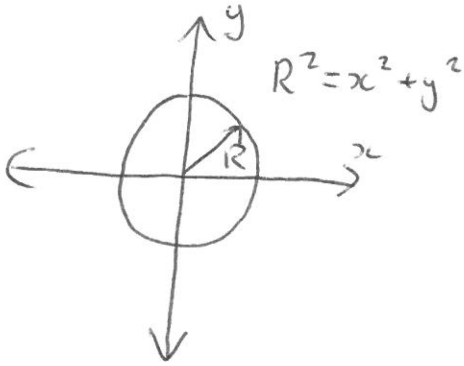
Woche 3

Aufgabe 1

$$A = \int_{x=-R}^{x=R} \int_{y=-\sqrt{R^2-x^2}}^{y=\sqrt{R^2-x^2}} 1 \, dy\, dx$$



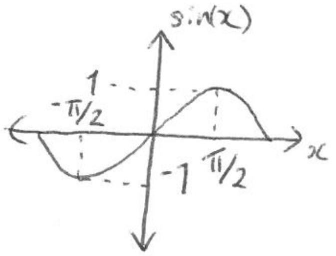
$$= \int_{-R}^{R} 2\sqrt{R^2-x^2}\, dx$$

$$= \left[x\sqrt{R^2-x^2} + R^2 \arcsin\left(\frac{x}{R}\right)\right]_{x=-R}^{x=R}$$

$$= \left(\cancel{R\sqrt{0}} + R^2 \arcsin(1)\right) - \left(\cancel{R\sqrt{0}} + R^2 \arcsin(-1)\right)$$

$$= R^2\left(\arcsin(1) - \arcsin(-1)\right)$$

$$= \underline{\pi R^2}$$



Aufgabe 2

$$A = \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} 1 \cdot r\, d\varphi \cdot dr$$

$$= 2\pi \left[\frac{r^2}{2}\right]_{r=0}^{r=R}$$

$$= \underline{\pi R^2}$$

Aufgabe 3

$$Q = \iiint dq$$

$$= \iiint \rho \, dVol$$

$$= \int_{z=0}^{z=L} \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} \frac{Az}{r}\, dV$$

$$= \int_{z=0}^{z=L} \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} \frac{Az}{\cancel{r}} \cancel{r}\, d\varphi\, dr\, dz$$

$$= 2\pi R A \left[\frac{z^2}{2}\right]_{z=0}^{z=L}$$

$$= \underline{\pi L^2 R A}$$

Aufgabe 4

$dq = \rho\, dV$

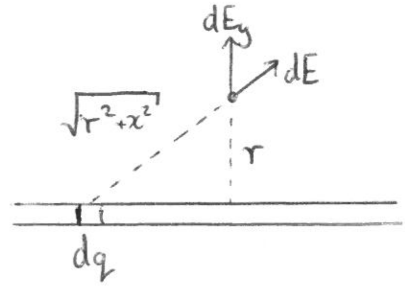
$dE = \dfrac{dq}{4\pi\varepsilon_0 (r^2+x^2)}$ (Coulombsches Gesetz)

$\dfrac{dE_y}{dE} = \dfrac{r}{\sqrt{r^2+x^2}}$ (Geometrie)

$\lambda = \dfrac{Q}{L} \Rightarrow \lambda = \dfrac{dq}{dx} \Rightarrow dq = \lambda\, dx$

$E_x = 0$ (Symmetrie)

$$E_y = \int dE_y$$



$$= \int \frac{r}{\sqrt{r^2+x^2}}\, dE$$

$$= \int \frac{r}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}\, dq$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{r}{(r^2+x^2)^{3/2}}\, dx$$

$$\boxed{= \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r}}$$