

Woche 3: Lösungen

1. Differentiation eines Vektors nach einem Parameter

$$1. a) \vec{r}(t) = \begin{pmatrix} 1+2t \\ 3t^2 \\ \frac{1}{2\sqrt{t}} \end{pmatrix}, \quad \dot{\vec{r}}(t) = \begin{pmatrix} 2 \\ 6t \\ -\frac{1}{4t^{3/2}} \end{pmatrix}$$

$$b) \vec{r}(t) = \begin{pmatrix} \cos t - t \sin t \\ 2t \sin t + t^2 \cos t \\ \frac{1}{3t^{2/3}} \end{pmatrix} \quad \dot{\vec{r}}(t) = \begin{pmatrix} -\sin t - \sin t - t \cos t \\ 2 \sin t + 2t \cos t + 2t \cos t - t^2 \sin t \\ -\frac{2}{9t^{5/3}} \end{pmatrix}$$

$$= \begin{pmatrix} -2 \sin t - t \cos t \\ (2-t^2) \sin t + 4t \cos t \\ -\frac{2}{9t^{5/3}} \end{pmatrix}$$

$$c) \vec{r}(t) = \begin{pmatrix} 2e^{2t} \\ 2te^{t^2} \\ 1 \end{pmatrix} \quad \dot{\vec{r}}(t) = \begin{pmatrix} 4e^{2t} \\ 2te^{t^2} + 4t^2e^{t^2} \\ 0 \end{pmatrix} = \begin{pmatrix} 4e^{2t} \\ 2te^{t^2}(1+2t) \\ 0 \end{pmatrix}$$

2. Funktion in Parameterdarstellung:

- Linie oder Ebene im 3D-Raum

2. Gradient eines Skalarfeldes

$$1) a) \frac{\partial \varphi}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}} \Rightarrow \nabla \varphi = \frac{\vec{r}}{|\vec{r}|}; \quad \vec{r} = \vec{x} + \vec{y} + \vec{z}$$

$$b) \frac{\partial \varphi}{\partial x} = e^{z^2-y}, \quad \frac{\partial \varphi}{\partial y} = -xe^{z^2-y}, \quad \frac{\partial \varphi}{\partial z} = 2xz e^{z^2-y}$$

$$\nabla \varphi = e^{z^2-y} (\vec{e}_x - x \vec{e}_y + 2xz \vec{e}_z)$$

